Euler Lagrange based Solution for Image Processing

Asha Ashok, Swathy Sekhar, Shajeesh K.U, Vidya M. K. P. Soman
Centre for Excellence in Computational Engineering and Networking
Amrita Vishwa Vidyapeetham, Ettimadai, Coimbatore, India
doi: 10.6088/ijaser.0020101026

Abstract: Image processing is one of the rapidly growing technologies with operations that can be applied to image data which may be in the form of a 2D, 3D or 4D signals. It has created tremendous opportunities for mathematical modeling, analysis, and computation. A novel approach in image processing has been developed, known as PDE (Partial Differential Equations) based image processing. The PDE approach is very promising for solving many problems in image processing because it provides new and more intuitive mathematical models, gives better approximations to the Euclidean geometry of the problem, and is supported by efficient discrete numerical algorithms based on difference approximations. In this paper, we have discussed how Euler Lagrange Equation plays a significant role in solving PDE based image processing problems. The typical image processing problems such as denoising and inpainting can be solved using EL equation. The ROF and Tikhonov models of denoising and Total Variation based inpainting has been implemented and a comparative study is done with the conventional methods. We have considered various well-known objective quality evaluation metrics used in image processing in this comparison. The results show that PDE based image processing techniques outperforms the conventional approaches.

Key words: ROF model; Tikhonov model; Partial Differential Equations; Image Denoising; Gradient flow.

1. Introduction

A new area of image processing, variational or PDE (Partial Differential Equations) based image processing has been evolved from the mathematical community by a group of new generation mathematicians in the recent years (Gilles Aubert and Pierre Kornprobst, 2006). PDE based image processing is different from conventional image processing techniques in that the processing is framed as a diffusion process or an optimization problem which can be solved using PDE techniques (Rafael Gonzalez and Richard Woods, 2004). In conventional linear methods of image processing, the operator is predefined and local information from the image cannot be used for finding the solution of the problem. In contrast, the nonlinear PDE based methods adjust the operation adaptively based on the local information in the image. For example, in case of image denoising, the conventional methods not only suppress the high frequency noise, but also smooth out the edges that play a significant role in the image analysis. The PDE based techniques can effectively remove noise while preserving edges in the image (Nadernejad and Hassanpour, 2005). Thus, the proposed method gives a simple and perfect mathematical depiction of the problem on a continuous domain.

Most of the optimization problems can be formulated as a partial differential equation using Euler Lagrange's equation. It is the fundamental equation of calculus of variations and can be defined as the differential equation whose solutions are the functions for which a given functional is stationary (Karras and Mertziou, 2004). Using EL Equation we derive a PDE for the optimal solution of the given variational formulation which is usually a minimization of the energy function over time.

The paper is organized as follows. The second section describes the Euler Lagrange equation and its formulation. In this section we describe two important applications of EL equation, image denoising and
image inpainting with necessary formulations. The fourth section covers the experimental results and analysis. The MatLab code for EL based Image denoising and inpainting is implemented and tested with standard image database. Five well known quality metrics are implemented. Based on this quality metric the performance of the PDE based method is evaluated. This section also covers the comparative study of PDE based methods against the classical methods based on the obtained performance metric. The conclusion is given in section five.

2. Theory

EL equation gives solution to obtain the maxima or minima of a functional of the form,

\[ I[y(x)] = \int_{x_1}^{x_2} F(x, y(x), y'(x))dx. \]  

(1)

It states that if \( J \) is defined by a functional \( J = I[y(x)] = \int_{x_1}^{x_2} F(x, y(x), y'(x))dx \) then \( J \) has a stationary value if the Euler-Lagrange differential equation,

\[ \frac{\partial F}{\partial y} - \frac{d}{dx}\left( \frac{\partial F}{\partial y'} \right) = 0 \]

(2)

is satisfied. EL equation is mainly used for solving optimization problem which involves minimizing or maximizing some functional (Keonwook Kang et al., 2006).

EL equation is only a necessary condition for the existence of the maxima or minima of an optimization problem. However, in many cases, the EL equation by itself is enough to give a complete solution of the problem. The EL equation is widely applied in the area of image denoising and inpainting. The first section deals with the formulation of EL equation. The next section focus on the applications of EL equation along with the formulations and the third section gives a brief description about the quality measures used for evaluation.

2.1 Formulation of EL equation

In calculus of variation, a Functional is of the form,

\[ I[y(x)] = \int_{x_1}^{x_2} F(x, y(x), y'(x))dx \]

(3)

Here, our objective is to find a function \( y(x) \) that minimizes (or maximizes) this functional, subject to certain boundary conditions, such as \( y(x_1) = a \) and \( y(x_2) = b \). For a functional \( I[y(x)] \), the first variation \( \delta I \) denotes the change in \( I \) if its argument \( y(x) \) changes by an infinitesimal amount \( \delta y(x) \). This is illustrated in figure 1, where function \( y(x) \) is shown in bold line (K. P. Soman and Ramanathan, 2012).
Euler Lagrange based Solution for Image Processing

Figure 1: Function $y(x)$ (bold line) and a small variation from it, $\tilde{y}(x) = y(x) + \delta y(x)$.  

$\delta y(x)$ is a small, continuous function which satisfies the boundary conditions $\delta y(x_1) = \delta y(x_2) = 0$.  

If the function undergoes a small change, $\tilde{y}(x) = y(x) + \delta \tilde{y}(x)$ where $\delta y(x)$ is a small, continuous function, then the variation of the functional is,  

$$\delta I = [I(\tilde{y}(x))] - [I(y(x))]$$  

Our task is to obtain the explicit expression of $\delta I$.  

To obtain $\delta I$, let us define  

$$\delta y(x) = \varepsilon \phi(x)$$  

where $\phi(x)$ is an arbitrary continuous function whose value is of the order 1 and $\varepsilon$ is an infinitesimal number. We also require $\phi(x)$ to be sufficiently smooth so that $\tilde{y}(x)$ is well defined.

Applying Taylor series expansion on $I[\tilde{y}(x)]$ in terms of $\varepsilon$,  

$$I[\tilde{y}(x)] = \int_{x_1}^{x_2} F(x, y(x), \varepsilon \phi(x), \delta y(x)) dx$$  

$$= \int_{x_1}^{x_2} \left[ F(x, y(x), \varepsilon \phi(x)) + \frac{\partial F}{\partial y} \varepsilon \phi(x) + \frac{\partial F}{\partial \phi} \delta y(x) + O(\varepsilon^2) \right] dx$$  

$$= I[y(x)] + \varepsilon \int_{x_1}^{x_2} \left[ \frac{\partial F}{\partial y} \phi(x) + \frac{\partial F}{\partial \phi} \phi(x) \right] dx + O(\varepsilon^2)$$  

Keeping terms only up to the first order, we have,  

$$\delta I = \varepsilon \int_{x_1}^{x_2} \left[ \frac{\partial F}{\partial y} \phi(x) + \frac{\partial F}{\partial \phi} \phi(x) \right] dx$$  

$$\delta I = \int_{x_1}^{x_2} \left[ \frac{\partial F}{\partial y} \delta y(x) + \frac{\partial F}{\partial y} \delta \tilde{y}(x) \right] dx$$  

(8)  

Usually, it is more desirable to express $\delta I$ solely in terms of $\delta y(x)$, and get rid of $\delta \tilde{y}(x)$ in the expression. This can be done by integration-by-parts.  

$$\delta I = \varepsilon \int_{x_1}^{x_2} \frac{\partial F}{\partial y} \phi(x) dx + \varepsilon \int_{x_1}^{x_2} \frac{\partial F}{\partial y} d\phi(x)$$  

$$= \varepsilon \int_{x_1}^{x_2} \frac{\partial F}{\partial y} \phi(x) dx + \varepsilon \left[ \frac{\partial F}{\partial y} \phi(x) \right]_{x_1}^{x_2} - \varepsilon \int_{x_1}^{x_2} \frac{d}{dx} \left( \frac{\partial F}{\partial \phi} \right) \phi(x) dx$$  

If we constrain the value of function $y(x)$ at $x_1$ and $x_2$ to be constant, i.e., apply boundary conditions $\phi(x_1) = \phi(x_2) = 0$, then
\[
\delta I = \int_a^b \left[ \frac{\partial F}{\partial y} - \frac{d}{dx} \left( \frac{\partial F}{\partial y'} \right) \right] \phi(x) dx
\]
\[= \int_a^b \left[ \frac{\partial F}{\partial y'} - \frac{d}{dx} \left( \frac{\partial F}{\partial y} \right) \right] \delta y(x) dx
\]
(9)

Therefore, if \( y(x) \) is a minimum (or a maximum) of functional \( I[y(x)] \), subjected to the constraint that \( y(x_1) \) and \( y(x_2) \) are fixed, then \( y(x) \) must satisfy the differential equation,
\[
\frac{\partial F}{\partial y} - \frac{d}{dx} \left( \frac{\partial F}{\partial y'} \right) = 0
\]
(10)

for \( x \in (x_1, x_2) \). This is the Euler-Lagrange equation for one dimensional signal. This can be extended to two dimension as
\[
\frac{\partial F}{\partial u} - \left( \frac{\partial}{\partial x} \frac{\partial F}{\partial u_x} + \frac{\partial}{\partial y} \frac{\partial F}{\partial u_y} \right) = 0
\]
(11)

2.2 Applications of EL equation

2.2.1 Image Denoising

Image denoising (Rudin et al., 1992) is a classical research problem in which a variety of conceptual frameworks and computational tools have been used widely. Denoising is the process of removing noise or blurs from the image, thereby improving the quality of a degraded picture. PDE based denoising methods preserves sharp edges which results in high quality image restoration. Three most popular variational methods for denoising are Tikhnov, ROF and TV-L1 model.

2.2.1.1 Rudin Osher and Fatemi (ROF Model)

Rudin, Osher and Fatemi (ROF) were the first who introduced Total Variation methods to Computer Vision in their paper on edge preserving image denoising (Rafael Gonzalez and Richard Woods, 2004). The model is designed to remove noise and other unwanted fine scale details, while preserving sharp discontinuities (edges). The ROF model (Chan et al, 1999) is defined as the constrained optimization problem.

\[
\min_{u} \left\{ \int_{\Omega} |\nabla u| d\Omega \right\} \quad \text{s.t} \quad \int_{\Omega} (u - f)^2 d\Omega = \sigma^2
\]
(12)

where \( f \) is the observed image function which is corrupted by Gaussian noise of variance \( \sigma^2 \) and \( u \) is the unknown denoised image.

In Nadernejad and Hassanpour, 2005, Chambolle showed that the original non-convex ROF model (Equation (12)) can be turned into a convex problem by replacing the equality constraint
\[
\int_{\Omega} (u - f)^2 d\Omega = \sigma^2
\]
by the inequality constraint \( \int_{\Omega} (u - f)^2 d\Omega \leq \sigma^2 \) which in turn can further be transformed to the unconstrained (or Lagrangian) model

\[
\min_{u} \left\{ \int_{\Omega} |\nabla u| d\Omega + \frac{1}{2\lambda} \int_{\Omega} (u - f)^2 d\Omega \right\}. \quad (13)
\]

where \( \lambda \) is the Lagrange multiplier.

The solution again can be found out by gradient flow.
Let $\min_u \{ E(u) \} = \int_{\Omega} |\nabla u(x,y)| d\Omega + \frac{1}{2\lambda} \int_{\Omega} |u-f|^2 d\Omega$

Let $F = |\nabla u(x,y)| + \frac{1}{2\lambda} (u-f)^2 = (u_x^2 + u_y^2)^{1/2} + \frac{1}{2\lambda} (u-f)^2$

That is, $F(u,u_x,u_y) = \frac{1}{2\lambda} (u-f)^2 + \sqrt{u_x^2 + u_y^2}$

Euler Lagrange equation for (13) is given by

$$\frac{\partial F}{\partial u} = \frac{\partial}{\partial x} \left( \frac{\partial F}{\partial u_x} \right) + \frac{\partial}{\partial y} \left( \frac{\partial F}{\partial u_y} \right) = 0$$

$$\frac{\partial}{\partial x} \left( \frac{\partial F}{\partial u_x} \right) = \frac{u_x \sqrt{u_x^2 + u_y^2} - u_x \sqrt{u_x^2 + u_y^2}}{u_x^2 + u_y^2}$$

Similarly $\frac{\partial}{\partial y} \left( \frac{\partial F}{\partial u_y} \right) = \frac{u_y u_x^2 - u_x u_y u_x + u_y u_x^2}{(u_x^2 + u_y^2)^{3/2}}$

So we obtain the EL equation as,

$$(u-f) - \frac{1}{\lambda} \frac{u_x u_x^2 - u_x u_y u_y + u_y u_x^2}{(u_x^2 + u_y^2)^{3/2}} = 0 \quad \text{(14)}$$

Visualizing $\frac{\partial F}{\partial u} \left( \frac{\partial}{\partial x} \frac{\partial F}{\partial u_x} + \frac{\partial}{\partial y} \frac{\partial F}{\partial u_y} \right)$ as gradient of the functional E, we represent it as $\nabla E$. Equating $\nabla E$ to time margin condition, we obtain the equilibrium solution as

$$\frac{\partial u}{\partial t} = -\nabla E \quad \text{(15)}$$

Substituting in (14), we get

$$\frac{\partial u}{\partial t} = - \left( \frac{\partial F}{\partial u} \left( \frac{\partial}{\partial x} \frac{\partial F}{\partial u_x} + \frac{\partial}{\partial y} \frac{\partial F}{\partial u_y} \right) \right) = (f-u) + \frac{1}{\lambda} \frac{u_x u_x^2 - u_x u_y u_y + u_y u_x^2}{(u_x^2 + u_y^2)^{3/2}} = 0$$

This is a sort of ‘Heat equation’ which tries to converge on a steady state solution.

### 2.2.1.2 Tikhonov model

This model (Chan et al, 1999) is formulated as L₂ norm optimization problem. The minimization problem is given by

$$\min_u \{ E(u) \} = \frac{1}{2} \int_{\Omega} |\nabla u(x,y)|^2 d\Omega + \frac{1}{2\lambda} \int_{\Omega} (f-u)^2 d\Omega \quad \text{(16)}$$

where the first term is the regularization term and the second term acts as the data fidelity term.

Applying the Euler Lagrange equation for the Tikhonov model,
Euler Lagrange based Solution for Image Processing

Let \( F = \frac{1}{2} \left[ \nabla u(x, y) \right]^2 + \frac{1}{2\lambda} (f - u)^2 = \frac{1}{2} (u_x^2 + u_y^2) + \frac{1}{2\lambda} (f - u)^2 \)

Euler lagrange equation is given by

\[
\frac{\partial F}{\partial u} - \left( \frac{\partial}{\partial x} \frac{\partial F}{\partial u_x} + \frac{\partial}{\partial y} \frac{\partial F}{\partial u_y} \right) = \frac{1}{\lambda} (f - u) - (u_{xx} + u_{yy}) = 0
\]

In Tikhonov model, since the regularization term is \( L_2 \) norm, we logically obtain blurry images, which do not correspond to pictures we can naturally observe.

2.2.2 Image Inpainting

It is the technique of modifying an image in an undetectable form to an observer not familiar with the original image (Tony Chan et al, 2002). Often times images may have regions with missing data. Examples may include scratches on images, the occlusion of objects in an image, or removing water marks from the images. To remedy these problems, image inpainting which is the filling of missing data based on known information is considered.

Given an image \( R \) and a region \( \Omega \) inside it, the inpainting problem consists of modifying the image values of the pixels in \( \Omega \) so that this region does not stand out with respect to its surroundings. The region \( \Omega \) is always given by the user for any inpainting problem (Chan et al., 1993). To inpaint an image we require image as well as the mask, \( \chi \) which is the region to be inpainted. Here we solve the inpainting optimization problem using Total Variation minimization.

2.2.2.1 Total Variation Inpainting

TV minimization is applied to inpainting process so that the missing or damaged region is filled in completely to model the appropriate geometric image features (Katsaggelos et al., 1993). Moreover, the TV minimization can systematically reduce the noise in images. The optimization problem is formulated as

\[
E(u) = \lambda \int_R \chi(u - u_0)^2 dxdy + \int_R |\nabla u| dxdy \tag{17}
\]

Where \( \chi(x, y) = \begin{cases} 0, & \text{if } (x, y) \text{ is part of point to be inpainted} \\ 1, & \text{if } (x, y) \text{ is part of points to be untouched} \end{cases} \)

\[
F(u, u_x, u_y) = \lambda \chi(u - u_0)^2 + \sqrt{u_x^2 + u_y^2}
\]

\[
\frac{\partial F}{\partial u} = 2\lambda \chi(u - u_0)
\]

\[
\frac{\partial F}{\partial u_x} = \frac{\partial}{\partial u_x} \left( \sqrt{u_x^2 + u_y^2} \right) = \frac{1}{2} \frac{2u_x}{\sqrt{u_x^2 + u_y^2}} = \frac{u_x}{\sqrt{u_x^2 + u_y^2}}
\]

\[
\frac{\partial F}{\partial u_y} = \frac{\partial}{\partial u_y} \left( \sqrt{u_x^2 + u_y^2} \right) = \frac{1}{2} \frac{2u_y}{\sqrt{u_x^2 + u_y^2}} = \frac{u_y}{\sqrt{u_x^2 + u_y^2}}
\]

\[
\frac{\partial}{\partial x} \frac{\partial F}{\partial u_x} = \frac{u_x}{\sqrt{u_x^2 + u_y^2}} - u_x \frac{\partial}{\partial x} \frac{1}{\sqrt{u_x^2 + u_y^2}} = \frac{u_x u_y^2 - u_x u_x u_y}{(u_x^2 + u_y^2)^{3/2}}
\]

Similarly \( \frac{\partial}{\partial y} \frac{\partial F}{\partial u_y} = \frac{u_y u_x^2}{(u_x^2 + u_y^2)^{3/2}} \)

Substituting in Euler Lagrange equation (11), we obtain
\[ \frac{\partial u}{\partial t} = \frac{u_x u_x^2 - u_y u_y u_{xy} + u_y u_y^2}{(u_x^2 + u_y^2)^{3/2}} - 2 \cdot \lambda \cdot \chi (u - u_0) \]

3. Quality Evaluation Metrics

Objective measures are evaluated based on mathematical measures and represents the quality by comparing the original (clean) image and degraded (enhanced) image. In this study we have chosen four well-known objective measures such as Mean Square Error (MSE), SNR, PSNR, and Structural Similarity (SSIM) (Nadernejad and Hassanpour, 2005).

3.1 Mean Square Error (MSE) Metric

MSE metric is the simplest and widely used quality metric. It is the mean of the squared difference between original and denoised image. MSE is defined as

\[ MSE = \frac{1}{NM} \sum_{i=1}^{N} \sum_{j=1}^{M} (I_{original}(i, j) - I_{denoised}(i, j))^2 \]

Where \( I_{original} \) is the original image and \( I_{denoised} \) is the denoised image. Smaller the MSE value, the better is the denoising performance.

3.2 Signal to Noise Ratio (SNR) Metric

SNR is the log of the ratio of signal variance to the noise signal variance and is defined as

\[ SNR_{db} = 10 \log_{10} \left( \frac{\sigma_{signal}^2}{\sigma_{noise}^2} \right) \]

Large SNR value of the enhanced image shows that the denoised image is closer to the original image and it poses a better quality.

3.3 Peak Signal to Noise Ratio (PSNR) Metric

PSNR, is the ratio of the maximum possible power of a signal to the power of corrupting noise. PSNR is usually expressed in terms of the logarithmic decibel scale. It is defined as,

\[ PSNR = 20 \log_{10} \left( \frac{MAX_I}{MSE} \right) \]

Here, \( MAX_I \) is the maximum possible pixel value of the image and MSE is mean squared error.

3.4 Structural Similarity (SSIM) Metric

SSIM measure gives the similarity between two images based on an initial clean image as reference. The SSIM metric is defined as,

\[ SSIM(x, y) = \frac{(2\mu_x \mu_y + c_1)(2\sigma_{xy} + c_2)}{\mu_x^2 + \mu_y^2 + \sigma_{xy}^2 + c_1(c_1 + c_2)} \]

where \( \mu_x \) is the average of \( x \), \( \mu_y \) is the average of \( y \), \( \sigma_x^2 \) is the variance of \( x \), \( \sigma_y^2 \) the variance of \( y \), \( \sigma_{xy} \) is the covariance of \( x \) and \( y \) and \( c_1 \) and \( c_2 \) are two variables to stabilize the division with weak denominator.
4. Results and Discussion

The two methods presented in second section for image denoising along with inpainting is implemented and tested with standard test images in matlab. The test images used in this experiment are ‘Lena.png’, ‘Boat.png’ and ‘Cameraman.tif’. Various parameter values in these three algorithms such as lambda the control parameter, step size, time limit etc. are adjusted in such a way that the quality of the enhanced image is improved considerably. Also objective quality evaluation is performed for the test images and comparative study of all the above applications of EL equations were performed based on these results. It is found that the ROF model performs better than the Tikhnov model for image denoising.

4.1 Image denoising

Salt & Pepper noise is added to the test images with an intensity of 0.01. In case of ROF model, lambda values are taken in a range 5 to 50. A clear denoised image is obtained when lambda is 50. The image becomes blurred for higher values of lambda. The objective measures PSNR, SNR, MSE and SSIM corresponding to ROF model are evaluated for different test images and is illustrated in the table 1.

![Figure 2: ROF denoising of ‘boat.png’ for λ=50](image)

![Figure 3: ROF denoising of ‘cameraman.tif’ for λ=50](image)

<table>
<thead>
<tr>
<th>Image</th>
<th>Lambda</th>
<th>SNR</th>
<th>PSNR</th>
<th>SSIM</th>
<th>MSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lena</td>
<td>5</td>
<td>13.7749</td>
<td>32.1354</td>
<td>0.9618</td>
<td>37.894</td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>13.5335</td>
<td>32.973</td>
<td>0.9546</td>
<td>30.2714</td>
</tr>
<tr>
<td></td>
<td>40</td>
<td>13.2887</td>
<td>32.6162</td>
<td>0.9463</td>
<td>32.8631</td>
</tr>
<tr>
<td></td>
<td>50</td>
<td>13.0804</td>
<td>32.6166</td>
<td>0.9445</td>
<td>32.8606</td>
</tr>
<tr>
<td>Cameraman</td>
<td>5</td>
<td>14.2954</td>
<td>30.6525</td>
<td>0.9058</td>
<td>55.0791</td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>14.3617</td>
<td>29.5604</td>
<td>0.8786</td>
<td>71.556</td>
</tr>
<tr>
<td></td>
<td>40</td>
<td>14.3862</td>
<td>29.2606</td>
<td>0.8645</td>
<td>75.8893</td>
</tr>
<tr>
<td></td>
<td>50</td>
<td>14.3857</td>
<td>29.0604</td>
<td>0.8627</td>
<td>79.4703</td>
</tr>
<tr>
<td>Boat</td>
<td>5</td>
<td>15.815</td>
<td>30.9567</td>
<td>0.9583</td>
<td>52.1689</td>
</tr>
</tbody>
</table>

Table 1: Objective quality measures for different input images with varying lambda in case of ROF model
The feasible range of lambda for better performance of Tikhonov model is found to be 0.4 to 0.8. The value of lambda varies depending on the image characteristics. When compared to ROF model, performance of this method is not satisfactory. The objective values for this model are also evaluated and the results are shown in table 2. The output of above two methods for the test images are shown in figures 1-3.

Table 2: Objective quality measures for different input images with varying lambda in case of Tikhonov model

<table>
<thead>
<tr>
<th>Image</th>
<th>Lambda</th>
<th>SNR</th>
<th>PSNR</th>
<th>SSIM</th>
<th>MSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lena</td>
<td>5</td>
<td>13.6736</td>
<td>29.0954</td>
<td>0.8831</td>
<td>73.925</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>13.071</td>
<td>30.4438</td>
<td>0.9152</td>
<td>54.1938</td>
</tr>
<tr>
<td></td>
<td>0.5</td>
<td>12.9188</td>
<td>29.1742</td>
<td>0.9076</td>
<td>72.5952</td>
</tr>
<tr>
<td></td>
<td>0.6</td>
<td>13.008</td>
<td>29.6332</td>
<td>0.9147</td>
<td>65.315</td>
</tr>
<tr>
<td></td>
<td>0.45</td>
<td>12.9492</td>
<td>28.9512</td>
<td>0.9068</td>
<td>76.4201</td>
</tr>
<tr>
<td>Cameraman</td>
<td>0.5</td>
<td>13.4667</td>
<td>24.0279</td>
<td>0.7707</td>
<td>253.1903</td>
</tr>
<tr>
<td></td>
<td>0.8</td>
<td>13.7941</td>
<td>25.1033</td>
<td>0.7956</td>
<td>197.5687</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>13.9271</td>
<td>25.6251</td>
<td>0.8052</td>
<td>175.2777</td>
</tr>
<tr>
<td>Boat</td>
<td>0.1</td>
<td>14.7949</td>
<td>23.6823</td>
<td>0.7295</td>
<td>278.5159</td>
</tr>
<tr>
<td></td>
<td>0.5</td>
<td>14.9196</td>
<td>26.9529</td>
<td>0.8759</td>
<td>131.1554</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>14.9875</td>
<td>28.5115</td>
<td>0.9049</td>
<td>91.6067</td>
</tr>
</tbody>
</table>

On comparison with the conventional methods of denoising, it is evident from the objective measures in table 3 that EL based denoising performs much better than existing filter based methods.

Table 3: Objective measures listed for Conventional based filtering methods

<table>
<thead>
<tr>
<th>Image</th>
<th>SNR</th>
<th>PSNR</th>
<th>SSIM</th>
<th>MSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Boat</td>
<td>15.2598</td>
<td>5.3764</td>
<td>0.0037</td>
<td>1.89E+04</td>
</tr>
<tr>
<td>Lena</td>
<td>13.5405</td>
<td>5.3436</td>
<td>0.0041</td>
<td>1.75E+04</td>
</tr>
<tr>
<td>Cameraman</td>
<td>14.3037</td>
<td>5.5475</td>
<td>0.0101</td>
<td>1.78E+04</td>
</tr>
</tbody>
</table>
4.2 Image Inpainting

To inpaint an image, the algorithm takes two inputs, the original image and a mask. Mask is an image which is having same size as the original image and it depicts the region to be inpainted in black pixels and the surrounding region in white pixels. The control parameter, lambda and time limit is adjusted in such a manner to obtain the desired results. For the test images used, the optimal value of lambda is found to be 5.

![Figure 5: Text removal from the image using Total Variation Inpainting (a) Original Image (b) Inpainted Image](image)

![Figure 6: Object removal from image by Inpainting (λ=5) (a) Original Image (b) Inpainted Image](image)

5. Conclusion

In this study, various methods of PDE-based image denoising and inpainting have been analyzed. In the analysis, various well-known measuring metrics have been used for evaluation of results. PDE based methods gives a much better result than the conventional filtering methods. In case of denoising, ROF model outperforms the Tikhnov model. The conventional methods of denoising which use filters reduce noise at the cost of smoothing the image and hence softening the edges whereas the PDE based methods take in to account the local information from the image and effectively remove noise while preserving edges in the image. When the energy of noise in an image is unpredictable, image denoising using EL equation is recommended. Experiments were conducted on various standard images for image inpainting with PDE approach and desired results were obtained.

6. References


