Common Fixed Points of Weak Compatible Mappings of Type (A)

in PM-Spaces

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Abstract: In this paper we prove common fixed point Theorem for six mappings in probabilistic metric space. Our result extends and generalizes some known results in metric space and probabilistic metric spaces.

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1. Introduction

The concept of probabilistic metric spaces (statistical metric spaces) was introduced initially by Menger (1942) which is the generalization of metric space. Since then, to use this concept in topology and analysis many authors have expansively developed the theory of probabilistic metric spaces. Especially, Schweizer-Sklar (1960,1983), Egbert (1968), Hadzic (1979), Serstnev (1963), Sherwood (1966,1971) and Stojakovic (1987,1988).

Recently, some fixed point theorems in probabilistic metric spaces have been proved by many authors; Bharucha-Reid (1976), Bocsan (1974), Cain and Kasriel(1976), Chang (1983), Cho, Murthy Stojakovic(1992), Ciric (1975), Dedeic-Sarapa, (1988), Hadzic (1978,1981), Hicks (1983), Singh-Pant (1984,1985), Stojakovic. Since every metric space is a probabilistic metric space and hence we can use many results in probabilistic metric spaces to prove some fixed point theorems in metric spaces and Banach spaces.

Sessa(1982) introduced the concept of weakly commuting mappings and Jungck (1986) introduced the concept of compatible mappings in metric spaces. In fact, weakly commuting mappings are compatible, but neither implication is Jungck-Murthy, Cho (1993) and Cho-Murthy Stojakovic (1992) introduced the concept of compatible mappings of type (A) on metric spaces and Menger spaces respectively and proved some common fixed point theorems for compatible mappings of type (A) on Menger space. In 1995, Pathak, Kang and Beak introduced the concept of weak compatible mapping of type (A) and proved some common fixed point theorems for weak compatible mappings of type (A) on Menger spaces. The aim of this paper is to prove common fixed point theorem for six mappings in probabilistic metric space. Our main result extends and generalize results of Pathak, Kang, Beak (S1995) and Cho, Murthy, Stojakovic (1992).

2. Preliminaries

Definition 2.1. Let R denote the set of reals and R* the non-negative reals. A mapping \( F : R \rightarrow R^* \) is

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called a distribution function if it is non-decreasing left continuous with \( \inf F = 0 \) and \( \sup F = 1 \).

**Definition 2.2.** A probabilistic metric space (briefly, a PM-space) is a pair \((X, F)\), where \(X\) is a non-empty set and \(F\) is a mapping from \(X \times X\) to \(L\), the set of all distribution functions. For \((u, v) \in X \times X\), the distribution function \(F(u, v)\) is denoted by \(F_{uv}\). The functions \(F(u, v)\) are assumed to satisfy the following condition :

(i) \(F(u, v; x) = 1\) for every \(x > 0\) if and only if \(u = v\),
(ii) \(F(u, v; 0) = 0\) for every \(u, v \in X\),
(iii) \(F(u, v; x) = F(v, u; x)\) for every \(u, v \in X\),
(iv) If \(F(u, v; x) = 1\) and \(F(v, w; y) = 1\), then

\[
F(u, w; x+y) = 1 \quad \text{for every } u, v, w \in X.
\]

In a metric space \((X, d)\), the metric \(d\) induces a mapping \(F : X \times X \rightarrow L\) such that

\[
F(u, v; x) = F_{uv}(x) = H(x - d(u, v))
\]

for every \(u, v \in X\) and \(x \in \mathbb{R}\), where \(H\) is the distribution function defined as \(H(x) = 0\) for \(x < 0\), \(1\) for \(x > 0\).

**Definition 2.3.** A map \(\Delta : [0, 1] \times [0, 1] \rightarrow [0, 1]\) is called a t-norm, if \(\Delta\) satisfies the following condition :

(i) \(\Delta (a, 1) = a\), \(\Delta (0, 0) = 0\);
(ii) \(\Delta (a, b) = \Delta (b, a)\);
(iii) \(\Delta (c, d) \geq \Delta (a, b)\) for \(c \geq a, d \geq b\);
(iv) \(\Delta (\Delta (a, b), c) = \Delta (a, \Delta (b, c))\).

**Example (i)\** \(\Delta(a, b) = ab\), (ii) \(\Delta(a, b) = \min(a, b)\), (iii) \(\Delta(a, b) = \max(a+b-1; 0)\).

**Definition 2.4.** A Menger space is a triplet \((X, F, \Delta)\) where \((X, F)\) is a PM-space and \(\Delta\) is a t-norm with the following condition

\[
F(u, w; x+y) \geq \Delta(F(u; v; x), F(v; w; y))
\]

The above inequality is called Menger’s triangle inequality.

**Example.** Let \(X = \mathbb{R}\), \(\Delta(a, b) = \min(a, b)\) for all \(a, b \in [0, 1]\) and

\[
F(u, v; x) = \begin{cases} 
H(x) & \text{for } u \neq v \\
1 & \text{for } u = v 
\end{cases}
\]

where \(H(x) = \begin{cases} 
0 & x \leq 0 \\
x & 0 \leq x \leq 1 \\
1 & x \geq 1 
\end{cases}\)

Then \((X, F, \Delta)\) is a Menger space.

Schweizer and Sklar [20] introduced the concept of neighbourhoods and convergence in PM-spaces as follows.

**Definition 2.5.** Let \((X, F, \Delta)\) be a Menger space. If \(u \in X\), \(\epsilon > 0\), \(\lambda \in (0, 1)\), then an \((\epsilon, \lambda)\) neighbourhood of \(u\), denoted by \(U_{\epsilon}(u, \lambda)\) is defined as

\[
U_{\epsilon}(u, \lambda) = \{ v \in X; F(u, v; \epsilon) > 1-\lambda \}.
\]

If \((X, F, \Delta)\) is a Menger space with the continuous t-norm \(\Delta\), then the family \(\{U_{\epsilon}(u, \lambda); u \in X, \epsilon > 0, \lambda \in (0, 1)\}\) of neighbourhoods induces a Hausdorff topology on \(X\) and if \(\sup_{a \in \mathbb{R}} \Delta(a, a) = 1\), it is metrizable.
Definition 2.6. A sequence \( \{p_n\} \) in \((X, F, \Delta)\) is said to be convergent to a point \( p \in X \) if for every \( \varepsilon > 0 \) and \( \lambda > 0 \), there exists an integer \( N = N(\varepsilon, \lambda) \) such that \( p_n \in U_p(\varepsilon, \lambda) \) for all \( n \geq N \) or equivalently \( F(p_m, p; \varepsilon) > 1 - \lambda \) for all \( n \geq N \).

Definition 2.7. A sequence \( \{p_n\} \) in \((X, F, \Delta)\) is said to be Cauchy sequence if for every \( \varepsilon > 0 \) and \( \lambda > 0 \), there exists an integer \( N = N(\varepsilon, \lambda) \) such that \( F(p_m, p_n; \varepsilon) > 1 - \lambda \) for all \( n, m \geq N \).

Definition 2.8. A Menger space \((X, F, \Delta)\) with the continuous t-norm \( \Delta \) is said to be complete if every Cauchy sequence in \( X \) converges to a point in \( X \).

Lemma 1 (1985). Let \( \{p_n\} \) be a sequence in a Menger space \((X, F, \Delta)\) where \( \Delta \) is continuous and \( \Delta(x, x) \geq x \) for all \( x \in [0,1] \). If there exists a constant \( k \in (0,1) \) such that for all \( x > 0 \) and \( n \in N \), \( F(p_n, p_{n+1}; kx) \geq F(p_{n-1}, p_n; x) \), then \( \{p_n\} \) is a Cauchy sequence.

Lemma 2 (1972). If \((X, d)\) is a metric space, then the metric \( d \) induces a mapping \( F : X \times X \to L \), defined by \( F(p, q) = H(x-d(p, q)), p, q \in X \) and \( x \in \varepsilon \). Further if \( \Delta : [0,1] \times [0,1] \to [0,1] \) is defined by \( \Delta(a,b) = \min\{a,b\} \), then \((X, \Delta)\) is a Menger space. It is complete if \((X, d)\) is complete. The space \((X, F, \Delta)\) so obtained is called the induced Menger space.

Remark 1 [1995]. The conditions “The T-norm \( \Delta \) is continuous and \( \Delta(x,x) \geq x \) for all \( x \in [0,1] \)” can be replaced by “\( \Delta(x,y) = \min\{x,y\} \) for all \( x, y \in [0,1] \)”. In fact, since \( \Delta(a,1) = a \) and \( \Delta(1,b) = b \) for all \( a, b \in [0,1] \), we have \( \Delta(a,b) \leq \min\{\Delta(a,1), \Delta(1,b)\} = \min\{a,b\} \) for all \( a, b \in [0,1] \).

On the other hand, we have
\[
\Delta(a,b) \geq \Delta(\min\{a,b\}, \min\{a,b\}) \geq \min\{a,b\}
\]
which implies \( \Delta(a,b) = \min\{a,b\} \).

3. Weak compatible mappings of type (A)

In this section, we give the concept of weak compatible mappings of type (A) in probabilistic metric spaces and some properties of these mappings for our main result.

Definition 3.1 (1991). Let \((X, F, \Delta)\) be a Menger space such that t-norm \( \Delta \) is continuous and \( S, T \) be mappings from \( X \) into itself. \( S \) and \( T \) are said to be compatible if
\[
\lim_{n \to \infty} F(STx_n, TSx_n; x) = 1
\]
for all \( x > 0 \), whenever \( \{x_n\} \) is a sequence in \( X \) such that
\[
\lim_{n \to \infty} Sx_n = \lim_{n \to \infty} Tx_n = z \text{ for all } z \in X.
\]

Definition 3.2 (1992). Let \((X, F, \Delta)\) be a Menger space such that t-norm \( \Delta \) is continuous and \( S, T \) be mappings from \( X \) into itself. \( S \) and \( T \) are said to be compatible of type (A) if
\[
\lim_{n \to \infty} F(STx_n, TTx_n; x) = 1 \text{ and } \lim_{n \to \infty} F(TSx_n, SSx_n; x) = 1
\]
for all \( x > 0 \), whenever \( \{x_n\} \) is a sequence in \( X \) such that
\[
\lim_{n \to \infty} Sx_n = \lim_{n \to \infty} Tx_n = z \text{ for some } z \in X.
\]

Definition 3.3 (1995). Let \((X, F, \Delta)\) be a Menger space such that the T-norm \( \Delta \) is continuous and \( S, T \) be mappings from \( X \) into itself. \( S \) and \( T \) are said to be weak compatible of type (A) if
\[
\lim_{n \to \infty} F(STx_n, TTx_n; x) \geq \lim_{n \to \infty} F(TSx_n, TTx_n; x)
\]
and
\[
\lim_{n \to \infty} F(TSx_n, SSx_n; x) \geq \lim_{n \to \infty} F(STx_n, SSx_n; x)
\]
for all $x > 0$, whenever $\{x_n\}$ is a sequence in $X$ such that 
\[ \lim_{n \to \infty} S x_n = \lim_{n \to \infty} T x_n = z \quad \text{for some } z \in X. \]

**Proposition 3.1** (1995). Let $(X, F, \Delta)$ be a Menger space such that the t-norm $\Delta$ is continuous and $\Delta(x, x) \geq x$ for all $x \in [0,1]$ and $S, T : X \to X$ be continuous mappings.

1. $S$ and $T$ are compatible if and only if they are compatible of type (A).
2. $S$ and $T$ are compatible of type (A) if and only if they are weak compatible of type (A).
3. $S$ and $T$ are compatible if and only if they are weak compatible of type (A).

**Proposition 3.2** (1995). Let $(X, F, \Delta)$ be a Menger space such that the t-norm $\Delta$ is continuous and $\Delta(x, x) \geq x$ for all $x \in [0,1]$ and $S, T : X \to X$ be mappings. If $S$ and $T$ are weak compatible of type (A) and $Sz = Tz$ for some $z \in X$, then 
\[ STz = TTz = TSz = SSz. \]

**Proposition 3.3** (1995). Let $(X, F, \Delta)$ be a Menger space such that the t-norm $\Delta$ is continuous and $\Delta(x, x) \geq x$ for all $x \in [0,1]$ and $S, T : X \to X$ be mappings. Let $S$ and $T$ be weak compatible mappings of type (A) and 
\[ \lim_{n \to \infty} S x_n = \lim_{n \to \infty} T x_n = z \quad \text{for some } z \in X. \]

Then we have
\[ (1) \quad \lim_{n \to \infty} TS x_n = Sz \quad \text{if } S \text{ is continuous at } z. \]
\[ (2) \quad STz = TSz \quad \text{and } Sz = Tz \quad \text{if } S \text{ and } T \text{ are continuous at } z. \]

### 4. Common fixed point theorems

In this section, we prove a common fixed point theorem for six mappings satisfying some conditions.

**Theorem 4.1.** Let $(X, F, \Delta)$ be a complete Menger space where $\Delta$ is continuous and $\Delta(x, y) = \min \{x, y\}$ for all $x, y \in [0,1]$ and $A, B, S, T, P$ and $Q$ be mappings from $X$ into itself such that

\[ (4.1) \quad P(X) \subseteq ST(X), Q(X) \subseteq AB(X), \]
\[ (4.2) \quad AB = BA, ST = TS, PB = BP, QS = SQ, QT = TQ \]
\[ (4.3) \quad \text{If one of } A, B, S, T, P \text{ and } Q \text{ are continuous}, \]
\[ (4.4) \quad \text{The pairs } (P, AB) \text{ and } (Q, ST) \text{ are weak compatible of type (A)}, \]
\[ (4.5) \quad \text{There exist a number } k \in (0,1) \text{ such that} \]
\[ [F(Px, Qy; kt)]^2 \geq \min\{[F(ABx, STy; t)]^2, F(ABx, Px; t). F(STy, Qy; t), \]
\[ F(ABx, STy; t). F(ABx, Px; t), F(STy, Qy; t). F(STy, Px; t), F(STy, Px; t). F(STy, Qy; t), \]
\[ F(ABx, Qy; 2t). F(STy, Px; t), F(STy, Px; t). F(STy, Px; t), F(STy, Qy; t). F(STy, Px; t), \]
\[ F(STy, Px; t). F(STy, Qy; t) \}, \]

for all $x, y \in X$ and $t \geq 0$.

Then $A, B, S, T, P$ and $Q$ have a unique common fixed point in $X$.

**Proof.** By (4.1), since $P(X) \subseteq ST(X)$, for any arbitrary $x_0 \in X$, there exists a point $x_1 \in X$ such that $Px_0 = STx_1$. Since $Q(X) \subseteq AB(X)$, for this point $x_1$, we can choose a point $x_2 \in X$ such that $Qx_1 = ABx_2$ and so on. Inductively, we can define a sequence $\{y_n\}$ in $X$ such that
\[ \begin{align*}
  y_{2n} &= Px_{2n} = STx_{2n+1} \quad \text{and } y_{2n+1} = Qx_{2n+1} = ABx_{2n+2} \\
  \text{for } n &= 0, 1, 2, \ldots
\end{align*} \]

Now, we shall prove $F(y_{2n}, y_{2n+1}; kt) \geq F(y_{2n-1}, y_{2n}; t)$ for all $t > 0$, where $k \in (0,1)$. Suppose that $F(y_{2n}, y_{2n+1}; kt) < F(y_{2n-1}, y_{2n}; t)$. Then by using (4.5) and $F(y_{2n}, y_{2n+1}; kt) \leq F(y_{2n}, y_{2n+1}; t)$, we have
\[ [F(y_{2n}, y_{2n+1}; kt)]^2 = [F(Ax_{2n}, Qx_{2n+1}; kt)]^2 \geq \min\{[F(ABx_{2n}, STx_{2n+1}; t)]^2, F(ABx_{2n}, Px_{2n}; t). F(STx_{2n+1}, Qx_{2n+1}; t), F(ABx_{2n}, STx_{2n+1}; t). F(ABx_{2n}, Px_{2n}; t), F(ABx_{2n}, STx_{2n+1}; t). F(ABx_{2n}, STx_{2n+1}; t) \}. \]
F(STx_{2n+1},Qx_{2n+1};t),F(ABx_{2n},STx_{2n+1};t),F(ABx_{2n},Qx_{2n+1};t),F(ABx_{2n},STx_{2n+1};t),F(STx_{2n+1},P_{2n};t),F(ABx_{2n},STx_{2n+1};t),F(STx_{2n+1},P_{2n};t),F(ABx_{2n},Qx_{2n+1};t),2t),F(STx_{2n+1},P_{2n};t),F(ABx_{2n},STx_{2n+1};t),F(STx_{2n+1},P_{2n};t),F(ABx_{2n},Qx_{2n+1};t),2t),F(STX_{2n+1},Qx_{2n+1};2t),F(STX_{2n+1},Qx_{2n+1};2t),
\geq \min \{ (F(2n-1,y_{2n+1};t),F(y_{2n-1},y_{2n+1};t),F(y_{2n-1},y_{2n+1};t),F(y_{2n-1},y_{2n+1};t),2t),F(y_{2n-1},y_{2n+1};t),F(y_{2n-1},y_{2n+1};t),F(y_{2n-1},y_{2n+1};t),2t),F(y_{2n-1},y_{2n+1};t),F(y_{2n-1},y_{2n+1};t),F(y_{2n-1},y_{2n+1};t),F(2n-1,y_{2n+1};t)\}
\geq \min \{ (F(2n-1,y_{2n+1};t)^2,F(y_{2n-1},y_{2n+1};t),F(y_{2n-1},y_{2n+1};t),F(y_{2n-1},y_{2n+1};t),2t),F(y_{2n-1},y_{2n+1};t),F(y_{2n-1},y_{2n+1};t),F(y_{2n-1},y_{2n+1};t),2t),F(y_{2n-1},y_{2n+1};t),F(y_{2n-1},y_{2n+1};t),F(y_{2n-1},y_{2n+1};t),F(2n-1,y_{2n+1};t)\}
\geq \min \{ (F(2n-1,y_{2n+1};kt)^2,F(y_{2n-1},y_{2n+1};kt),F(y_{2n-1},y_{2n+1};kt),F(y_{2n-1},y_{2n+1};kt),2t),F(y_{2n-1},y_{2n+1};kt),F(y_{2n-1},y_{2n+1};kt),F(y_{2n-1},y_{2n+1};kt),F(2n-1,y_{2n+1};kt)\}
= F(y_{2n-1},y_{2n+1};kt)^2
\]
which is a contradiction. Thus, we have
F(y_{2n-1},y_{2n+1};kt) \geq F(y_{2n-1},y_{2n+1};t).

Similarly, we have also
F(y_{2n+1},y_{2n+2};kt) \geq F(y_{2n+1},y_{2n+1};t).

Therefore, for every x \in N, F(y_{2n+1},y_{2n+1};kt) \geq F(y_{2n-1},y_{2n+1};t).

Therefore, by Lemma (1), \{y_n\} is a Cauchy sequence in X. Since the Menger space (X, F, \Delta) is complete, \{y_n\} converge to a point z in X, and the subsequences \{P_{2n}\}, \{AB_{2n}\}, \{ST_{2n}\}, \{Q_{2n}\} of \{y_n\} also converge to z.

Now, suppose that P is continuous. Since P and AB are weak compatible of type (A), it follows from Proposition (3.3) that
PPx_{2n} \text{ and } (AB)P_{2n} \rightarrow Pz \text{ as } n \rightarrow \infty.

By (4.5), we have
\[\text{[F(PPx_{2n}, Qx_{2n+1};kt)]^2} \geq \min \{ (F(AB(Px_{2n}), STx_{2n+1};t), F(AB(Px_{2n}), P(Px_{2n};t), F(STx_{2n+1}, Qx_{2n+1};t), F(AB(Px_{2n}), STx_{2n+1};t), F(AB(Px_{2n}), P(Px_{2n};t), F(AB(Px_{2n}), STx_{2n+1};t), F(AB(Px_{2n}), Qx_{2n+1};2t), F(AB(Px_{2n}), STx_{2n+1};t), F(STx_{2n+1}, P(Px_{2n};t), F(AB(Px_{2n}), Qx_{2n+1};2t), F(STx_{2n+1}, P(Px_{2n};t), F(AB(Px_{2n}), P(Px_{2n};t), F(STx_{2n+1}, P(Px_{2n};t), F(AB(Px_{2n}), Qx_{2n+1};2t), F(STx_{2n+1}, P(Px_{2n};t), F(AB(Px_{2n}), Qx_{2n+1};2t)) \}
\]
Taking n \rightarrow \infty, we have
\[\text{[F(Pz, z;kt)]^2} \geq \min \{ (F(Pz, z;kt), F(Pz, Pz; t), F(z,z; t), F(Pz,z; t), F(Pz,Pz; t), F(Pz,Pz; t), F(Pz,z; t), F(Pz,z; t), F(Pz,z; t), F(Pz,z; t), F(Pz,z; t), F(Pz,z; t), F(Pz,z; t), F(Pz,z; t), F(Pz,z; t)) \]
\[= [F(Pz, z; x)]^2 \]
which is a contradiction. Thus, we have Pz = z. Since P(X) \subset ST(X), there exists a point u \in X such that z = Pz = STp. Again by (4.5), we have
Taking \( n \to \infty \), we obtain
\[
[F(x, Q_p; kt)]^2 \geq \min \{[F(P_{2n}, Q; t)]^2, F(P_{2n}, P_{2n}; t). F(z, Q_p; t), F(z, z; t)\},
\]
which implies that \( Q_p = z \). Since \( Q \) and \( ST \) are weak compatible of type (A) and \( Q_p = ST_p = z \) by Proposition (3.2), \( Q(ST)p = (ST)Qp \) and hence \( Qz = STz \). Moreover, by (4.5), we have
\[
[F(P_{2n}, x; 2t)]^2 \geq \min \{[F(AB_{2n}, ST; t)]^2, F(AB_{2n}, P_{2n}; t). F(S(Tz), Qz; t), F(AB_{2n}, ST; t). F(AB_{2n}, P_{2n}; t), F(AB_{2n}, Qz; 2t), F(AB_{2n}, ST; t). F(S(Tz), P_{2n}; t), F(AB_{2n}, Qz; 2t). F(S(Tz), P_{2n}; t), F(AB_{2n}, P_{2n}; t). F(S(Tz), P_{2n}; t), F(AB_{2n}, Qz; 2t). F(S(Tz), Qz; t)\},
\]
so that we have \( Pq = z \). Since \( P \) and \( AB \) are weak compatible of type (A) and \( Pq = ABq = z \), \( P(AB)q = (AB)Pq \) and hence \( Pz = ABz \). Now, we show that \( Tz = z \). By putting \( x = x_{2n} \), and \( y = Tz \) in (4.5) and using (4.2) we have
\[
[F(P_{2n}, Q(Tz); kt)]^2 \geq \min \{[F(AB_{2n}, ST(Tz); t)]^2, F(AB_{2n}, P_{2n}; t), F(AB_{2n}, ST(Tz); t). F(ST(Tz), Q(Tz); t), F(AB_{2n}, ST(Tz); t). F(ST(Tz), P_{2n}; t), F(AB_{2n}, Q(Tz); 2t), F(AB_{2n}, ST(Tz); t). F(ST(Tz), P_{2n}; t), F(AB_{2n}, Q(Tz); 2t). F(ST(Tz), P_{2n}; t)\},
\]
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letting $n \to \infty$, we have:

$$[F(z,Tz;kt)]^2 \geq \min\{[F(z,Tz;t)]^2, F(z,z;t), F(Tz,z;t), F(Tz,Tz;t), F(z,Tz;2t), F(z,Tz;ct), F(Tz,z;2t), F(Tz,Tz;t)\}.$$

so that we have $Tz = z$. Since $STz = z$, therefore, $Sz = z$. Finally, we show that $Bz = z$. By putting $x = Bz$ and $y = x^{2n+1}$ in (4.5) and using (4.2), we have:

$$[F(Bz,z;kt)]^2 \geq \min\{[F(Bz,z;t)]^2, F(Bz,Bz;t), F(z,z;t), F(Bz,z;2t), F(Bz,Bz;2t), F(Bz,z;2t), F(Bz,Bz;2t), F(Bz,z;ct), F(Bz,Bz;ct)\}.$$

Taking $n \to \infty$, we have:

$$[F(z,w;kt)]^2 \geq \min\{[F(z,w;t)]^2, F(z,z;t), F(w,w;t), F(z,w;t), F(w,z;t), F(z,w;2t), F(w,z;2t), F(z,w;2t), F(w,z;2t)\}.$$

Thus, we have $z = w$. This completes the proof of the Theorem.

If we put $B = T = I$ in Theorem 4.1, we have the following result:

**Cor 4.2.** Let $(X, F, \Delta)$ be a complete Menger space where $\Delta$ is continuous and $\Delta(x,y) = \min(x,y)$ for all $x, y \in [0,1]$. Let $A, S, P$ and $Q$ be mappings from $X$ into itself such that

1. $P(X) \subset A(X), Q(X) \subset S(X)$
2. The pairs $\{P, A\}$ and $\{Q, S\}$ are weak compatible of type (A)
3. One of $A, S, P$ and $Q$ is continuous.
4. There exists a number $k \in (0,1)$ such that

$$[F(Px,Qy;kt)]^2 \geq \min\{[F(Ax,Sy;t)]^2, F(Ax,Px;t), F(Sy,Qy;t), F(Ax,Sy;t), F(Ax,Px;t), F(Ax,Sy;t), F(Sy,Qy;t), F(Ax,Py;kt), F(Ax,Qy;2t), F(Ax,Py;kt), F(Ax,Py;kt)\}.$$
Common Fixed Points of Weak Compatible Mappings of Type (A) in PM-Spaces

F(Ax, Qy; 2t). F(Sy, Px; t), F(Ax, Px; t). F(Sy, Px; t), F(Ax, Qy; 2t). F(Sy, Qy; t)

for all x, y ∈ X and t > 0.

Then A, S, P and Q have a unique common fixed point in X.

If we put A = B = S = T = I in Theorem 4.1, we have the following:

**Cor. 4.3.** Let (X, F, Δ) be a complete Menger space where Δ is continuous and Δ(x, y) = min (x, y) for all x, y ∈ [0,1]. Let P, Q be weak compatible of type (A) on X into itself such that P(X) ⊂ Q(X). If one of P and Q is continuous mappings from X into itself and there exists a constant k ∈ (0,1) such that

\[ F(Px, Qy; kt)^2 \geq \min \{ F(x,y; t)^2, F(x, Px; t).F(y, Qy; t), F(x,y; t).F(x, Px; t), F(x,y; t).F(y, Px; t), F(x, Py; 2t).F(y, Px; t), F(x, Px; t).F(y, Px; t), F(x, Py; 2t), F(x, Px; t).F(y, Px; t).F(x, Py; 2t).F(y, Py; t) \}, \]

for all x, y ∈ X and t > 0.

Then P and Q have a unique common fixed point in X.

Now we give metric version of the Theorem 4.1.

**Theorem 4.2.** Let A, B, S, T, P and Q be mappings from a complete metric space (X, d) into itself such that

(4.10) P(X) ⊂ ST(X), Q(X) ⊂ AB(X)
(4.11) AB = BA, ST = TS, PB = BP, QS = SQ, QT = TQ
(4.12) If one of A, B, S, T, P and Q are continuous.
(4.13) The pairs {P, AB} and {Q, ST} are weak compatible of type (A).
(4.14) \[ d^2(Px, Qy) \leq k \max \{ d^2(ABx, STy), d(ABx, Px).d(STy, Qy), d(ABx, STy).d(ABx, Px), d(ABx, STy).d(STy, Qy), \]
\[ \frac{1}{2} d(ABx, STy).d(ABx, Qy), d(ABx, STy).d(STy, Px), \]
\[ \frac{1}{2} d(ABx, Qy), d(STy, Px), d(STy, Px), \]
\[ \frac{1}{2} d(ABx, Qy). d(STy, Qy) \}

for all x, y in X, where k ∈ (0,1).

Then A, B, S, T, P and Q have a unique common fixed point in X.

Which is direct consequences of the Theorem (4.1) and Lemma (2).

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**References**