Two commodity perishable inventory system with negative and impatient customers, postponed demands and a finite population

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Abstract: In this article, we consider a stochastic inventory system with two different items in stock, one is major item (first commodity) and other is gift item (second commodity), and the demands originate from a finite population of \(N\) sources. The maximum storage capacity for the \(i\)-th commodity is \(S_i\) \((i=1, 2)\) time points of demand occurrences form independent quasi random distributions. The second commodity is supplied as a gift whenever the demand occurs for the first commodity, but no major item (first commodity) is provided as a gift for demanding a second commodity. The items are perishable in nature. If the inventory position of first commodity is zero then any arriving demand for the first commodity either enters a pool or leaves the system according to a Bernoulli trial. The system has been identified as a Markov process and the system performance measures are derived. The total expected cost rate is also considered and the results are illustrated numerically.

Keywords: Compliment item, continuous time inventory system, postponed demand, negative customers, perishable inventory, finite population.

1. Introduction

Modeling and analysis of inventory systems with multiple items have been subject matter for many investigators in the past. Such studies vary from simple extensions of EOQ analysis to sophisticated stochastic models references may be found in Miller (1971), Agarwal (1984), Silver (1974), Thomstone and Silver (1975), Kalpakam and Arivarignan (1993) and Srinivasan and Ravichandran (1994) and the references therein. Various policies of placing orders in these systems have been proposed and studied in the literature. In continuous review inventory systems, Ballintify (1964) and Silver (1974) have considered a coordinated reordering policy which is represented by the triplet \((S, c, s)\), where the three parameters \(S_i\), \(c_i\) and \(s_i\) are specified for each item \(i\) with \(s_i \leq c_i \leq S_i\), under the unit sized Poisson demand and constant lead time. In this policy, if the level of \(i\)-th commodity at any time is below \(s_i\), an order is placed for \(S_i - s_i\) items and at the same time, any other item \(j\) \((\neq i)\) with available inventory at or below its can-order level \(c_j\), an order is placed so as to bring its level back to its maximum capacity \(S_j\). Subsequently many articles have appeared with models involving the above policy and another article of interest is due to Federgruen et al. (1984), which dealt with the general case of compound Poisson demands and non-zero lead times.

Anbazhagan and Arivarignan (2000, 2001 and 2003) have analyzed two commodity inventory system under various ordering policies. Yadavalli et al. (2004) have analyzed a model with joint ordering policy and varying order quantities. Yadavalli et al. (2006) have considered a two-commodity substitutable inventory system with Poisson demands and arbitrarily distributed lead time. In a very recent paper, Anbazhagan et al. (2011) considered analysis of two commodity inventory system with compliment for bulk demand in which, one of the items (say \(C_1\)) is designated as the major item and the other item (say \(C_2\))

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as the sub-item (gift item) is supplied whenever the demand for major item is greater than or equal to \( r \) (a specified number \( r > 0 \)). \((s, S)\) type policy for the major item, with random lead time but instantaneous replenishment for the gift item are considered. The lost sales for major item are also assumed when the items are out of stock.

All the models listed above assume lost sales of demands during the stock-out periods. In the case of backlogging, the backlogged demands are satisfied immediately when the ordered items are materialized. But in some real life situations the demand that has to be backlogged may have to wait even after the stock is replenished. This type of inventory problems are called inventory with postponed demands. The concept of postponed demands in inventory has been introduced by Berman et al. (1993) who have assumed that both demand and service rates are deterministic. Krishnamoorthy and Islam (2004) have considered a Markovian inventory system with exponential lead time and the pooled customers are selected according to an exponentially distributed time lag. Sivakumar and Arivarignan (2008) considered an inventory model in which the demand occurs according to a Markovian arrival process, lead time is distributed as phase type, exponential life time for the items in the stock and the pooled customers are selected exponentially. Paul Manuel et al. (2007) dealt an inventory system in which the positive and negative demand occurs according to independent Markovian arrival processes, lead time of the reorder, life time of the items, inter-selection time of a customers from the pool and the reneging time points of the customers in the pool are independent exponential distribution. Sivakumar and Arivarignan (2009) considered an inventory system with independent Markovian arrival processes for positive and negative demands, exponential lead time for the reorders, exponential life times for each items in the stock and the pool size is infinite. The customers are selected one-by-one according to the first come first serve discipline and the inventory level is above or equal to a prefixed level which is varied between one and the reorder level.

In the present paper, we assume that both commodities (major item and gift item) are sold separately, and as a gift for each major item a customer buys, a sub-item is supplied with, but not vice versa. The demands originate from a finite population of \( N \) sources and the items are perishable in nature. The demands that occur for first commodity during the stock out periods either enter a pool or leave the system. The demands in the pool are selected one-by-one, as long as the replenished stock is above \( s_1 \), with the interval time between any two successive selections is distributed as exponential. The joint probability distributions for both commodities and the number of customers in the pool are obtained in the steady state case. The measures of system performance in the steady state are calculated and the total expected cost rate is also considered. The results are illustrated numerically.

The rest of the paper is organized as follows. In section 2, we describe the mathematical model and the notations used in this paper are defined. The formulation and the analysis of the model are presented in section 3. The steady state probability vector of the Markov chain is calculated in section 4 and some key system performance measures are derived in section 5. Several numerical results that illustrate the influence of the system parameters on the system performance are discussed in section 6.

1.1 Mathematical models
We consider a stochastic inventory system with two different items in stock, one is major item (first commodity) and other is gift item (second commodity), and the demands originate from a finite population of N sources. Each source is either free or in the pool at any time. The time points of demand occurrences form independent quasi random distributions each with parameter $\lambda_i$ ($i=1,2$). If the demand occurs for first commodity then one unit of second commodity is supplied as a gift to the customer who ordered a unit of first commodity, but not vice versa i.e., no first commodity (major item) is supplied as a gift for ordering the second commodity. As and when the on-hand inventory level of first commodity drops to a prefixed level, $s_1(\geq 0)$ an order for $Q_1(=S_1-s_1)$ units is placed. The lead time of this order is exponentially distributed with parameter $\beta$ ($>0$). An $(0,S_2)$ ordering policy is adopted for the second commodity with zero lead time. The items are perishable in nature. The life time of each commodity is exponential with parameter $\gamma_i$ ($i=1,2$).

Any arriving demand for first commodity, when its inventory level is zero, is offered the choice of either leaving the system immediately or being postponed until the ordered items are received. We assume that the demanding customer accept the offer of postponement according to independent Bernoulli trials with probability $p$. With probability $q=1-p$, the customer decline the offer and is considered to be lost. After the replenishment and as long as the inventory level of first commodity is greater than $s_1$, the customers in the pool are selected one-by-one according to an exponentially distributed time lag between two successive selections with rate $\mu$ ($>0$). In addition to the regular demands of the first commodity, a second flow of negative customers following quasi random distribution with parameter $\nu$ ($>0$) is also considered. The negative demand will remove one of the demands waiting in the pool. The removal policy adopted in the paper is RCE (removal of a customer from the end of the queue). Further, we have assumed that an impatient customer in the pool leaves the system independently after a random time which is distributed as negative exponential with parameter $\alpha$ ($>0$). We have also assumed that the input flows of regular and negative demands, lead times of reorder, life time of each item, inter-selection time of customers from the pool and reneging times are mutually independent random variables.

2. Analysis

Let $L_1(t)$, $L_2(t)$ and $X(t)$ denote the inventory position of commodity-I, the inventory position of commodity-II and the number of customers in the pool respectively. From the assumptions made on the input and output processes it can be shown that triplet $\{(L_1(t),L_2(t),X(t)),t \geq 0\}$ is a continuous time Markov chain with state space given by $E$. By ordering the state space as lexicographically, the infinitesimal generator $\Theta = (a((i_1,i_2,i_3),(j_1,j_2,j_3)),(i_1,i_2,i_3),(j_1,j_2,j_3)) \in E$ and the $\Theta$ can be written in terms of submatrices $\Theta_{i_1,j_1}$, namely, $\Theta = ((\Theta_{i_1,j_1}))$, Where
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\[
[\Theta]_{i_1 j_1} = \begin{cases} 
A_{i_1}, & j_1 = i_1, \quad i_1 \in V_0^{s_1} \\
B_{i_1}, & j_1 = i_1 - 1, \quad i_1 \in V_1^{s_1} \\
C, & j_1 = i_1 + Q_1, \quad i_1 \in V_0^{s_1} \\
0, & \text{otherwise.}
\end{cases}
\]

With

\[
[C]_{2j_2} = \begin{cases} 
W, & j_2 = i_2, \quad i_2 \in V_1^{s_2} \\
0, & \text{otherwise.}
\end{cases}
\]

\[W = \beta I_{N+1}\]

For \( i_1 = 1, K, S_1 \)

\[
[B_{i_1}]_{2j_2} = \begin{cases} 
D_{i_1}, & j_2 = i_2, \quad i_2 \in V_1^{s_2} \\
0, & \text{otherwise.}
\end{cases}
\]

For \( i_1 = 1, 2, 3, K, S_1; \)

\[
[D_{i_1}]_{i_3 j_3} = \begin{cases} 
(N - i_3)\lambda_i + i_3\gamma_i, & j_3 = i_3, \quad i_3 \in V_0^{N-1} \\
T(i_3 - s_3)\lambda_i + \mu_i, & j_3 = i_3 - 1, \quad i_3 \in V_0^N \\
i_3\gamma_i, & j_3 = i_3, \quad i_3 = N \\
0, & \text{otherwise}
\end{cases}
\]

For \( i_1 = 0; \)

\[
[A_{i_1}]_{2j_2} = \begin{cases} 
F_{i_2}, & j_2 = S_2, \quad i_2 = 1 \\
F_{i_2}, & j_2 = i_2 - 1, \quad i_2 \in V_2^{s_2} \\
G_{i_2}, & j_2 = i_2, \quad i_2 \in V_1^{s_2} \\
0, & \text{otherwise.}
\end{cases}
\]

For \( i_2 = 1, 2, 3, K, S_2; \)

\[
[F_{i_2}]_{i_3 j_3} = \begin{cases} 
(N - i_3)\lambda_i + i_3\gamma_2, & j_3 = i_3, \quad i_3 \in V_0^N \\
0, & \text{otherwise.}
\end{cases}
\]

For \( i_2 = 1, 2, 3, K, S_2; \)
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\[
\begin{align*}
& \begin{cases}
  p(N - i_s) \lambda_i, & j_s = i_s + 1, \quad i_s \in V^{n-1}_0 \\
  \nu + i_s \alpha, & j_s = i_s, \quad i_s \in V^{n-1}_1 \\
  N \alpha, & j_s = i_s - 1, \quad i_s = N
\end{cases}
\end{align*}
\]

\[
|G_{1,2,3}|_{3,3} = 
\begin{cases}
  -((N - i_s)(p \lambda_i + \lambda_i \delta_{i,s}^N) + (N - i_s)w \delta_{i,s}^o + i_s \gamma_2 + i_s \alpha + \beta), & j_s = i_s, \quad i_s \in V^{n-1}_0 \\
  -i_s \gamma_2 + N \alpha + \beta), & j_s = i_s, \quad i_s = N \\
  0 & \text{otherwise}
\end{cases}
\]

For \( i_1 = 1, 2, 3, K, S_1; \)

\[
|A_{1,2}|_{1,2} = 
\begin{cases}
  F_{1,2} & j_2 = S_2, \quad i_2 = 1 \\
  F_{1,2} & j_2 = i_2 - 1, \quad i_2 \in V^{7}{2} \\
  H_{1,2} & j_2 = i_2, \quad i_2 \in V^{7}{2} \\
  0 & \text{otherwise}
\end{cases}
\]

For \( i_2 = 1, 2, 3, K, S_1; \) For \( i_2 = 1, 2, 3, K, S_2; \)

\[
|A_{2,1}|_{2,1} = 
\begin{cases}
  (N - i_s) \nu + i_s \alpha & j_s = i_s - 1, \quad i_s \in V^{7}_1 \\
  -((N - i_s)(\lambda_i + \lambda_i) \delta_{i,s}^N) + (N - i_s)w \delta_{i,s}^o + i_s \gamma_2 + i_s \alpha + \beta \langle s_i - i_1 \rangle + \langle t_i - s_i \rangle, & j_s = i_s, \quad i_s \in V^{n-1}_0 \\
  -i_s \gamma_2 + i_s \gamma_1 + N \alpha + \beta \langle s_i - i_1 \rangle + \langle t_i - s_i \rangle, & j_s = i_s, \quad i_s = N \\
  0 & \text{otherwise}
\end{cases}
\]

It may be noted that the matrices \( A_1, B_1, \quad i_1 = 1, 2, K, S_1, \quad A_0 \) and \( C \) are square matrices of order \( S_2(N + 1) \). The sub matrices \( W, D_1, F_{1,2}, G_{1,2}, H_{1,2}, \quad i_1 = 1, 2, K, S_1, \quad i_2 = 1, 2, 3, K, S_2; \) are square matrices of order \( (N + 1) \).
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2.1 Steady state analysis

It can be seen from the structure of $\Theta$ that the homogeneous Markov process
\[ \{(L_1(t), L_2(t), X(t)) : t \geq 0\} \] on the finite space $E$ is irreducible, aperiodic and persistent non-null.

Hence the limiting distribution
\[ \lim_{t \to \infty} P[ L_1(t) = i_1, L_2(t) = i_2, X(t) = i_3 | L_1(0), L_2(0), X(0)] \] exists.

Let $\Pi = (\Pi^{(0)}, \Pi^{(1)}, K, \Pi^{(S)})$.

Partitioning the vector, $\Pi^{(i)}$ into as follows:

\[ \Pi^{(i)} = (\Pi^{(i,1)}, \Pi^{(i,2)}, K, \Pi^{(i,3)}, i_1 = 0, 1, 2, K, S_1) \]
which is partitioned as follows:

\[ \Pi^{(i_1,i_2)} = (\Pi^{(i_1,i_2,0)}, K, \Pi^{(i_1,i_2,N)}) \]

Then the vector of limiting probabilities $\Pi$ satisfies

\[ \Pi \Theta = 0 \quad \text{and} \quad \Pi e = 1. \] (1)

The first equation of (1) yields the following set of equation

\[ \Pi^{(i_1+)} B_{q+1} + \Pi^{(i_1)} A_{q} = 0, i_1 = 0, 1, K, Q_1 - 1, \] (2)

\[ \Pi^{(i+)} B_{q+1} + \Pi^{(i)} A_{q} + \Pi^{(q-Q)} C = 0, i_1 = Q_1, \] (3)

\[ \Pi^{(q+)} B_{q+1} + \Pi^{(q)} A_{q} + \Pi^{(q-Q)} C = 0, i_1 = Q_1 + 1, K, S_1 - 1, \] (4)

\[ \Pi^{(q)} A_{q} + \Pi^{(q-Q)} C = 0, i_1 = S_1. \] (5)

The equations (except (3)) can be recursively solved to get

\[ \Pi^{(i)} = \Pi^{(Q)} \Omega_{q}, \quad i_1 = 0, 1, K, S_1, \]

Where
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\[
\Omega_{t_i} = \begin{cases} 
( -1 )^{Q_1 - i_1} B_{Q_1} A_{Q_1 - 1}^{-1} B_{Q_1 - 1} A_{Q_1 - 1}^{-1} \Lambda B_{i_1 + 1} A_{i_1}^{-1}, & i_1 = 0, 1, K , Q_1 - 1, \\
I, & i_1 = Q_1, \\
( -1 )^{2Q_1 - i_1 + 1} \sum_{j_1 = 0}^{S_1 - i_1} \left[ (B_{Q_1} A_{Q_1 - 1}^{-1} B_{Q_1 - 1} A_{Q_1 - 1}^{-1} \Lambda B_{s_1 - j_1 - 1} A_{s_1 - j_1 - 1}^{-1} ) C A_{s_1 - j_1}^{-1} \\
\times (B_{s_1 - j_1} A_{s_1 - j_1 - 1}^{-1} B_{s_1 - j_1 - 1} A_{s_1 - j_1 - 1}^{-1} \Lambda B_{i_1 + 1} A_{i_1}^{-1} ) \right] & i_1 = Q_1 + 1, K , S_1, 
\end{cases}
\]

and \( \Pi^{(Q_1)} \) can be obtained by solving

\[
\Pi^{(Q_1)} \left[ \begin{array}{c}
( -1 )^{Q_1} \sum_{j_1 = 0}^{S_1 - i_1} \left[ (B_{Q_1} A_{Q_1 - 1}^{-1} B_{Q_1 - 1} A_{Q_1 - 1}^{-1} \Lambda B_{s_1 - j_1 - 1} A_{s_1 - j_1 - 1}^{-1} ) C A_{s_1 - j_1}^{-1} \\
\times (B_{s_1 - j_1} A_{s_1 - j_1 - 1}^{-1} B_{s_1 - j_1 - 1} A_{s_1 - j_1 - 1}^{-1} \Lambda B_{i_1 + 1} A_{i_1}^{-1} ) \right] \\
B_{Q_1 + 1} + A_{Q_1} \\
+ ( -1 )^{Q_1} B_{Q_1} A_{Q_1 - 1}^{-1} B_{Q_1 - 1} A_{Q_1 - 1}^{-1} \Lambda B_{1} A_{0}^{-1} C \end{array} \right] = 0,
\]

and

\[
\Pi^{(Q_1)} \left[ \sum_{i_1 = 0}^{Q_1 - 1} ( -1 )^{Q_1 - i_1} B_{Q_1} A_{Q_1 - 1}^{-1} B_{Q_1 - 1} A_{Q_1 - 1}^{-1} \Lambda B_{i_1 + 1} A_{i_1}^{-1} \right] + I + \\
\sum_{i_1 = Q_1 + 1}^{S_1} \left[ ( -1 )^{2Q_1 - i_1 + 1} \sum_{j_1 = 0}^{S_1 - i_1} \left[ (B_{Q_1} A_{Q_1 - 1}^{-1} B_{Q_1 - 1} A_{Q_1 - 1}^{-1} \Lambda B_{s_1 - j_1 - 1} A_{s_1 - j_1 - 1}^{-1} ) C A_{s_1 - j_1}^{-1} \\
\times (B_{s_1 - j_1} A_{s_1 - j_1 - 1}^{-1} B_{s_1 - j_1 - 1} A_{s_1 - j_1 - 1}^{-1} \Lambda B_{i_1 + 1} A_{i_1}^{-1} ) \right] \right] = 1.
\]

3. System performance measures

In this section some performance measures of the system under consideration in the steady state are derived.

3.1 Expected inventory level

Let \( \eta_{t_1} \) and \( \eta_{t_2} \) denote the average inventory level for the first commodity and second
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commodity respectively in the steady state. Then we have,

\[ \eta_{I_1} = \sum_{i_1=1}^{S_1} \sum_{i_2=1}^{S_2} \sum_{i_3=0}^{N} i_1 \pi^{(i_1,i_2,i_3)} \]

and

\[ \eta_{I_2} = \sum_{i_1=0}^{S_1} \sum_{i_2=1}^{S_2} \sum_{i_3=0}^{N} i_2 \pi^{(i_1,i_2,i_3)} \]

3.2 Expected reorder rate

Let \( \eta_{R_1} \) and \( \eta_{R_2} \) denote the expected reorder rate for the first commodity and second commodity respectively in the steady state. Then we have,

\[ \eta_{R_1} = \sum_{i_2=1}^{S_2} \sum_{i_3=0}^{N} (\bar{\delta}_{y_0} (N - i_3) \lambda_1 + \bar{\delta}_{y_0} i_3 \mu + (s_1 + 1) \gamma_1) \pi^{(i_1,i_2,i_3)} \]

and

\[ \eta_{R_2} = \sum_{i_1=0}^{S_1} \sum_{i_3=0}^{N} (\bar{\delta}_{y_0} (N - i_3) \lambda_1 + \bar{\delta}_{y_0} i_3 \mu + \bar{\delta}_{y_0} (N - i_3) \lambda_2 + \gamma_2) \pi^{(i_1,i_3)} \]

3.3 Expected perishable rate

Let \( \eta_{P_1} \) and \( \eta_{P_2} \) denote the expected perishable rates for the first commodity and the second commodity respectively in the steady state. Then

\[ \eta_{P_1} = \sum_{i_1=1}^{S_1} \sum_{i_2=1}^{S_2} \sum_{i_3=0}^{N} i_1 \gamma_1 \pi^{(i_1,i_2,i_3)} \]

and

\[ \eta_{P_2} = \sum_{i_1=0}^{S_1} \sum_{i_2=1}^{S_2} \sum_{i_3=0}^{N} i_2 \gamma_2 \pi^{(i_1,i_2,i_3)} \]

3.4 Expected number of demands in the pool

Let \( \eta_{PC} \) denote the expected number of demands in the pool. Then

\[ \eta_{PC} = \sum_{i_1=0}^{S_1} \sum_{i_2=1}^{S_2} \sum_{i_3=1}^{N} i_3 \pi^{(i_1,i_2,i_3)} \]
3.5 Expected reneging rate

Let $\eta_{RR}$ denote the expected reneging rate of customers in the steady state. Then

$$
\eta_{RR} = \sum_{i_1=0}^{S_1} \sum_{i_2=1}^{S_2} \sum_{i_3=1}^{N} i_3 \alpha \pi^{(i_1,i_2,i_3)}
$$

3.6 Expected arrivals rate of negative customers

Let $\eta_{AN}$ denote the expected arrival rate of negative customers in the steady state. Then

$$
\eta_{AN} = \sum_{i_1=0}^{S_1} \sum_{i_2=1}^{S_2} \sum_{i_3=1}^{N-1} (N - i_3) \nu \pi^{(i_1,i_2,i_3)}
$$

3.7 Average customers lost to the system

Let $\eta_{AL}$ denote the average number of customers lost to the system. Then

$$
\eta_{AL} = \sum_{i_2=1}^{S_2} \sum_{i_3=0}^{N-1} q (N - i_3) \lambda_i \pi^{(0,i_2,i_3)}
$$

3.8 Mean Waiting Time

Let $\eta_W$ denote the mean waiting time of the customers in the pool. Then

$$
\eta_W = \frac{\eta_{PC}}{\eta_{AP}}
$$

Where $\eta_{PC}$ is the expected number of customers in the pool and the effective arrival rate (Ross [14]), $\eta_{AP}$ is given by

$$
\eta_{AP} = \sum_{i_2=1}^{S_2} \sum_{i_3=0}^{N-1} p (N - i_3) \lambda_i \pi^{(0,i_2,i_3)}
$$

3.9 Cost analysis

To compute the total expected cost per unit time (total expected cost rate), the following costs, are considered.

$c_{h1}$ : The inventory holding cost per unit item per unit time for I-commodity.

$c_{h2}$ : The inventory holding cost per unit item per unit time for II-commodity.

$c_{s1}$ : The setup cost per order of I-commodity.
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\( c_{s2} \) : The setup cost per order of II-commodity.

\( c_{p1} \) : Perishable cost of I-commodity per unit item per unit time.

\( c_{p2} \) : Perishable cost of II-commodity per unit item per unit time.

\( c_w \) : Waiting cost of an orbiting demand per unit time.

\( c_r \) : Reneging cost per customer per unit time.

\( c_n \) : Loss per unit time due to arrival of a negative customer.

\( c_l \) : Cost of a customer lost per unit time.

The long run total expected cost rate is given by

\[
TC(S_1, S_2, s_1, s_2, N) = c_{h1} \eta_{h1} + c_{h2} \eta_{h2} + c_{p1} \eta_{p1} + c_{p2} \eta_{p2} + c_w \eta_{pc} + c_r \eta_{rr} + c_n \eta_{an} + c_l \eta_{al}.
\]

Where \( \eta \)'s are as given in (5.1) - (5.8).

Due to the complex form of the limiting distribution, it is difficult to discuss the properties of the cost function analytically. Hence, a detailed computational study of the cost function is carried out.

4. Numerical Illustrations

In this section we discuss some interesting numerical examples that qualitatively describe the performance of the inventory model under study. Our experience with considerable numerical examples indicates the function \( TC(S_1, S_2) \), appropriate numerical search procedures are used to obtain the optimal values of \( TC, S_1 \) and \( s_1 \) (say \( TC^*, S_1^* \) and \( s_1^* \)). The effects of varying the system parameters and costs on the optimal values have been studied and the results agreed with what one would expect.

In Table 1 gives the total expected cost rate as a function of \( s_1^* \) and \( S_1^* \) by fixing the parameters: \( S_2 = 16, \ N = 10, \ \lambda_1 = 1, \ \lambda_2 = 4, \ \gamma_1 = 0.2, \ \gamma_2 = 0.1, \ \beta = 1, \ \alpha = 0.02, \ \nu = 0.01, \ \mu = 0.05, \ p = 0.8, \ c_{h1} = 0.001, \ c_{h2} = 0.001, \ c_{p1} = 0.1012, \ c_{p2} = 0.9, \ c_{sl} = 0.001, \ c_{p2} = 0.002, \ c_n = 0.008, \ c_r = 0.009, \ c_l = 0.001 \) and \( c_l = 0.009 \). From the Table 1 the total expected cost rate is more sensitive to the changes in \( s_1^* \) than that of in \( S_1^* \). Some of the results are presented in Tables 2 through 6 where the lower entry in each cell gives the total expected cost rate and the upper entries the corresponding \( S_1^* \) and \( s_1^* \).
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Next the impact of $c_{h1}$, $c_{h2}$, $c_{s1}$, $c_{s2}$, $c_{pl}$, $c_{p2}$, $c_w$, $c_r$, $c_n$ and $c_i$ on the optimal values $(s_1^*, S_1^*)$ and the corresponding $TC^*$ are studied by fixing the parameters: $\lambda_1 = 1$, $\lambda_2 = 4$, $\gamma_1 = 0.2$, $\lambda_1 = 1$, $\lambda_2 = 4$, $\gamma_1 = 0.2$, $\gamma_2 = 0.1$, $\alpha = 0.02$, $\nu = 0.01$, $\mu = 0.05$, $p = 0.8$.

When any one of the cost values $c_{h1}$, $c_{h2}$, $c_{s1}$, $c_{s2}$, $c_{pl}$, $c_{p2}$, $c_w$, $c_r$, $c_n$ and $c_i$ monotonically increases and other costs are fixed then the total expected cost rate monotonically increases. But, it does not change the optimal values $s_1^*$ and $S_1^*$ (see Tables 2 - 6).

Fixing all parameters and other cost values except $s_1^*$, the expected total cost rates are computed as shown in Table 7. The four curves in Fig. 1 correspond to $(S_1, S_2) = (30, 30)$, $(S_1, S_2) = (30, 25)$, $(S_1, S_2) = (25, 25)$ and $(S_1, S_2) = (25, 30)$ represent different convex functions of $s_1$.

Table 1: Total expected cost rate as a function of $S_1$ and $s_1$

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5. Conclusions

In this paper we consider a finite source two commodity perishable inventory system with compliment, postponed demands and negative customers. The joint probability distribution for both commodities and number of demands in the pool is obtained in the steady state case. Various system performance measures are derived and the long-run total expected cost rate is calculated. By assuming a suitable cost structure on the inventory system, we have presented extensive numerical illustrations to show the effect of change of values for constants on the total expected cost rate.

Table 2: Sensitivity of $c_{h1}$ and $c_{h2}$ on the optimal values
Two commodity perishable inventory system with negative and impatient customers, postponed demands and a finite population

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Two commodity perishable inventory system with negative and impatient customers, postponed demands and a finite population

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Table 5: Influence of $c_l$ and $c_w$ on the optimal values

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Table 6: Variation in optimal values for different values of $c_n$ and $c_r$

Table 7: Effect of $s_1$ values on Expected total cost rate
Two commodity perishable inventory system with negative and impatient customers, postponed demands and a finite population

\[
(s_1, s_2)
\]

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Figure 1: Effect of \(s_1\) values on Total Expected Cost Rate

Acknowledgement
K. Jeganathan’s research was supported by University Grants Commission of India under Rajiv Gandhi National Fellowship F.16-1574/2010(SA-III)

6. References


Two commodity perishable inventory system with negative and impatient customers, postponed demands and a finite population